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## COMPUTER SIMULATIONS OF CHARGED PARTICLE MOTION IN PULSED FIELDS

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### Introduction

Analysis of the interaction between electric and magnetic fields and particles moving in a vacuum is fundamental to many problems in electrical and electronic engineering. Analytical models have been developed for many traditional devices such as cathode ray and vacuum tubes which rely on these interactions for their operation. Recently, however, many new areas of research based on particle-field interactions have emerged. In the pulsed-power field these include fundamental studies of the interaction of high frequency fields and static or moving charges<sup>[1]</sup>, the synchronization of pulsed magnetrons for radar applications<sup>[2]</sup> and the development of new devices such as high-power microwave generating devices<sup>[3]</sup> or miniature vacuum devices based on silicon processing technology<sup>[4]</sup>.

The operation of many of these new devices has become complicated and may be dependent upon several parameters. The analytical solution of such problems is often intractable and numerical simulation therefore has an increasingly important role to play in optimisation and design.

Transmission line modelling (TLM) is a numerical analysis technique which has been applied extensively to the solution of electromagnetic field problems<sup>[5,6,7]</sup>. It is a time domain approach and as such is ideally suited to studies of pulsed and transient effects. In this paper we explain how moving or static charge particles can be incorporated in a two-dimensional TLM electromagnetic field model. The new model is validated by applying it to a simple vacuum diode for which analytical results are available for comparison purposes.

### Incorporation of Charged Particle Motion in Transmission Line Modelling

The solution of the electromagnetic field problem is based on a numerical two dimensional transmission line model (TLM) employing series connected nodes<sup>[7,8]</sup>. This discretizes the region to be studied into a mesh of step length  $\Delta L$  as illustrated for a simple planar vacuum diode in Figure 1. Time is stepped in intervals  $\Delta t$  where  $\Delta t = \Delta L/C$  is the time a wave takes to propagate from one node to an adjoining node. An electric field can be set up inside this region by applying the voltage source  $V_0$  to the anode. An additional contribution to the field arises from charges injected into the modelled region from the cathode.

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At a particular time instant the TLM is used to find the electric field at each mesh point in the usual way. Once the electric field and any applied z-directed magnetic field are known at each mesh point, the x and y components of the force on each charge can be calculated. For each modelled charge the computer program developed stores the charge size and its calculated velocity, exact position and nearest node position. A further field calculation need only be performed for the duration of any transient effects or when the applied voltage or the nearest-node position of any charge changes. Predictions of when these discretized charge positions will occur are automatically made within the computer program which makes the computational process very efficient. To model the movement of a charge Q from one mesh point to another a current I is allowed to flow between the two nodes considered for time  $kat$  where  $I = (Q/kat)$  and k is the number of time steps over which the charge is moved.

### Results

A diode with 3 cm wide plates separated by 1 cm was studied. The region between the plates was divided into a regular mesh with node separation 0.1 cm in both x and y directions. The width of each electrode was one node spacing. A uniform electric field was set up between the plates of the diode by application of a static voltage  $V_s$ . Charges were injected from the cathode to the first row of nodes to give an emission current density <sup>(9)</sup>.

$$J = A_0 T^2 \exp\left[-\frac{W}{kT}\right] \quad (1)$$

where  $A_0$  is a constant dependent on the cathode material, T is the assumed cathode temperature in degrees kelvin, k is Boltzmann's constant and W is the work function of the cathode material.

Because the electric field is only y directed the uniform injection taken effectively reduces the model to a one-dimensional one if no magnetic field forces are considered. The static diode current arising was measured in two ways; directly as the sum of the current flowing in the two wires A and B and as the rate of flow of charge over a given period at any mesh point. These computed currents were found to be identical in the steady state.

Figure 2 shows the calculated voltage distribution between the cathode and anode for an applied voltage of 50V with no electron emission and then emission corresponding to a tungsten cathode operating at temperatures of  $T_1 = 2000$  K and  $T_2 = 2150$  K. The curves show clearly the effect of space charge in lowering the potential and changing the sign of the electric field near the cathode for high injection and agree with theoretical predictions <sup>(9,10)</sup>. Figure 3 shows the I-V characteristic of a diode with a tungsten cathode operating at a temperature of 2000K. The space charge limited and saturation regions anticipated can be clearly seen. The saturation current  $I_{sat}$  was also found for several tungsten cathode temperatures in the range 1750-2250K. From the results obtained the straight line relation between  $I_{sat}/T^2$  and  $1/T$  (Equation 1) was plotted as shown in Figure 4. The slope of this plot produces the correct value for the work function of tungsten further validating the numerical analysis.

## Conclusion

Static or moving charged particles have successfully been incorporated in a transmission line model for numerical electromagnetic field computations based on a two-dimensional series connected node. The approach has been validated by applying it to a simple vacuum diode for which analytical results can be obtained. The time-domain nature of the transmission line approach makes the new computer program developed ideal for simulating pulsed power phenomena and devices.

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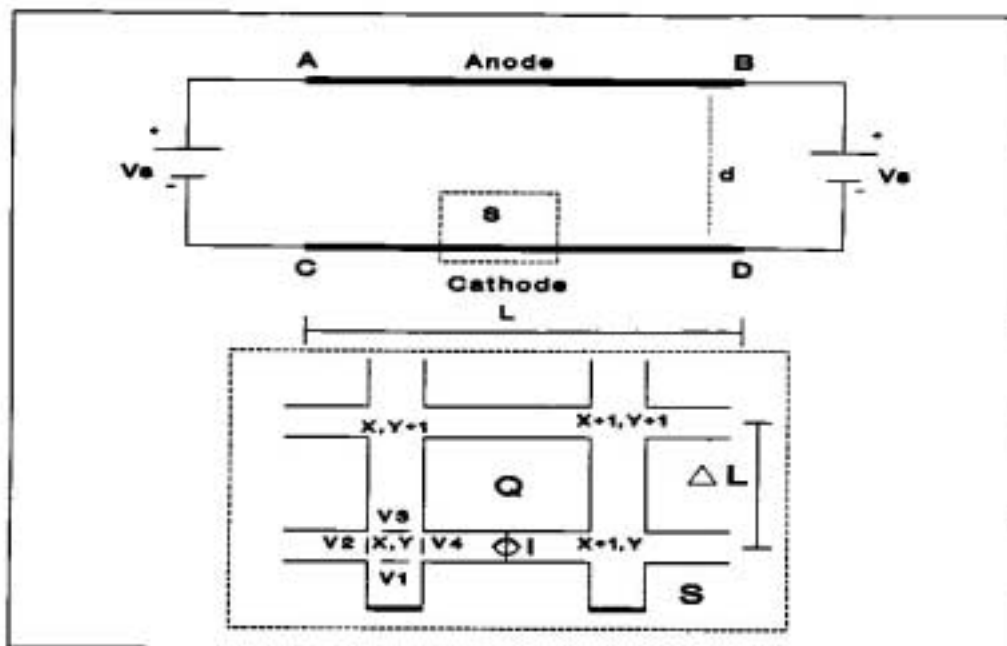


Fig.1 Discretization of Planar Vacuum diode into a two dimensional TLM model.

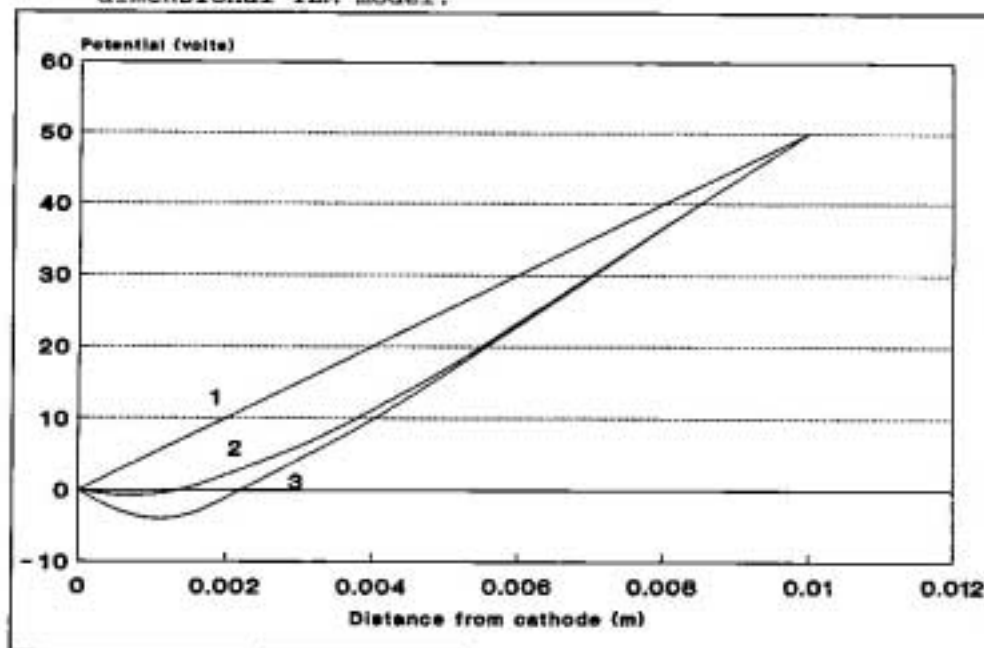


Fig.2 Variation of Potential with distance from the cathode for: 1) No emission, 2) cathode temperature  $T_1=2000$  K, 3) cathode temperature  $T_2=2150$  K.

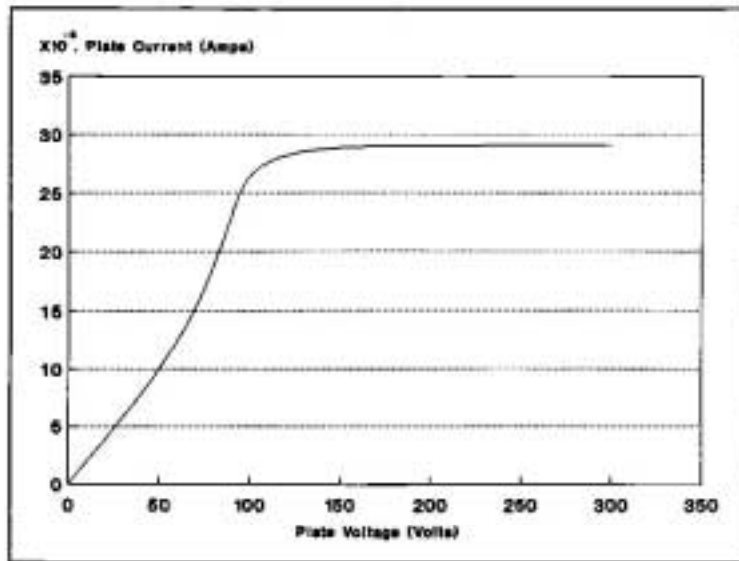


Fig.3 I-V Characteristic of a planar diode studied for a tungsten cathode operating at 2000 K.

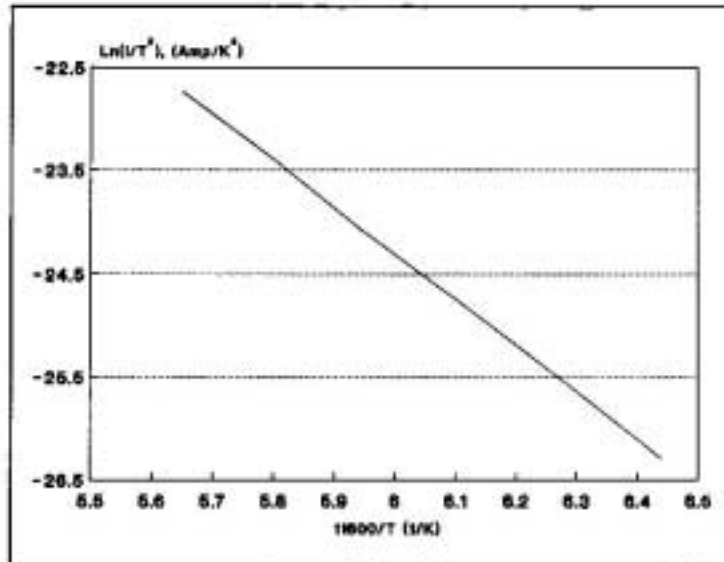


Fig.4 Plot of  $\ln(I_{sat}/T^3)$  against  $(1/T)$  for the planar diode studied.