Impedance Calculation of High Frequency Shielded Cables with the Aid of Conformal Mappings

Alexandros Gavrilakis[*] **Muhammed Al-Asadi** **Alistair P. Duffy** **Kenneth G. Hodge** **Arthur J. Willis**

School of Engineering and Technology
De Montfort University
The Gateway, Leicester, UK.
+44 116 257 7056 * alex@dmu.ac.uk

Brand-Rex Ltd.
Viewfield Industrial Estate
Glenrothes, Fife, Scotland

Abstract

This paper presents a new method for the calculation of the impedance of high frequency channels and in particular that of shielded twisted pair cables. Its main advantage is that it uses a more realistic shield geometry instead of the ideal cylindrical case, which most of the related literature is based on. It uses conformal mapping for the conductor-to-conductor capacitance and effective permittivity calculations for extra accuracy. Lastly, characteristic impedance equations are derived. Comparison with measurements is made to show the improvement; practical considerations of the new method are also addressed.

Keywords

Shielded cables, conformal mappings, impedance

1. Introduction

This paper is concerned with high frequency shielded channels. In this configuration two copper conductors, each enclosed in its own insulation are twisted together and are surrounded by a shield. The purpose of the shield is to add extra protection to the pair from electromagnetic interference initiated by external sources. Up to date, the highest frequency is achieved with Category 7 cables that can work up to frequencies of 600 MHz [1]. Significant work has been done to improve the accuracy of Unshielded Twisted Pair (UTP) Cable formulae [2]. However, in the case of Shielded Twisted Pair (STP) cables the literature relating to the formulation of primary parameters as capacitance and inductance of STP cables is more complex and a deeper understanding is required.

In the shielded case the complexity of the configuration increases dramatically, as the number of conducting surfaces from just two in the UTP case (two parallel conductors) increases to three (two parallel conductors and the shield) in the STP case. Based on the fact that the structure is usually symmetrical with respect to the centre of the shield, it means that only two capacitance values are required to describe the system. One is the mutual capacitance between the parallel conductors, the other is the capacitance between each individual conductor and the shield. The structure of the paper is as follows. Initially, the relations relating the electrical parameters of the conductors and the shield (charge, potential) are presented and are applicable to any given geometry subject to the determination of a number of constants. The major step then is to obtain these constants by analysing the geometrical structure of the system. Then, conformal transformations are used for the calculation of the mutual capacitance. Furthermore a method to calculate the effective permittivity is introduced. Finally, expressions for the characteristic impedance are developed and compared with measurements.

2. Capacitance calculation

2.1 Coefficients of potential

In a system consisting of two parallel conductors, surrounded by a grounded shield, as in Figure 1 the potential on the surface of each one,

$$\Phi_1 = r_{11} \cdot q_1 + r_{12} \cdot q_2 \quad (1)$$

$$\Phi_2 = r_{21} \cdot q_1 + r_{22} \cdot q_2 \quad (2)$$

where the r constants depend solely on the geometry of the system

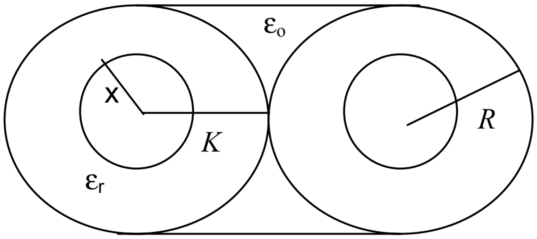

Figure 1: Practical cross-section shape of twisted-pair cable

In order to calculate the potentials Φ_1 and Φ_2 the determination of the r constants is required. First constant to be determined is r_{11}, which relates each conductor with the shield.

The first important point to observe is that the surfaces of the inner conductors, when they are not directly facing each other, they correspond to that ones of a coaxial case. This gives a potential of

$$\Phi_y = \frac{q_y \cdot \ln\left(\frac{R}{x}\right)}{\pi \cdot \varepsilon_r} \quad (3)$$

where R is the radius of the dielectric, x the radius of the inner conductor, ε_r the dielectric permittivity and y=1 or 2 (for each inner conductor).

For a coaxial system

$$C = \frac{q_y}{\Phi_y - \Phi_o} \tag{4a}$$

and $\Phi_o=0$ (grounded shield), then

$$C_A = \frac{\pi\varepsilon_r}{\ln\left(\frac{R}{x}\right)} \tag{4b}$$

The next point to calculate is the conductor to shield capacitance when the shield is not any more cylindrical, but can be represented satisfactory by a straight line.

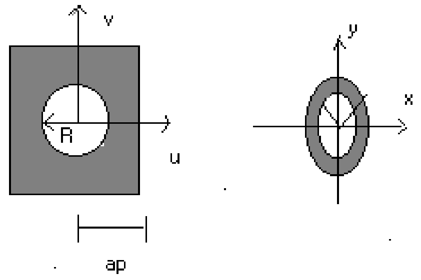

Fig. 2 Approximate conformal mapping of doubly connected region

The following conformal transformation [3], mapping polygons and circles in the w plane into approximate circles in the z plane, as in Figure 2, has been used to account, in part, for the shape of the shield.

$$w = \alpha_p A_p \sum_{j=0}^{\infty} \alpha_j \cdot z^{jp+1} \tag{5}$$

$$\alpha_j = \binom{-\frac{2}{p}}{j} \cdot \frac{1}{jp+1}(-1)^j \tag{6}$$

Where a_p is the apothem of the polygon, p is the number of axes of symmetry and A_p is a constant relating to p. In this method the values of p and A_p is 4 and 1.078 respectively [3].

$$w = \alpha_p\left(1.078z - 0.108z^5 + 0.045z^9 - 0.026z^{13} + \cdots\right) \tag{7}$$

Having converted this part of the shield into two eccentric circles, then the more simplified coaxial theory can be used. If the dielectric was homogeneous the capacitance C would be

$$C = \frac{\pi \cdot \varepsilon_r}{2 \cdot \ln\left(\frac{1.078 \cdot R}{x}\right)} \tag{8}$$

However because in a cable there is an air filling in the area where the dielectric is not present, then,

$$C_B = \frac{\dfrac{\pi \cdot \varepsilon_r}{2 \cdot \ln\left(\frac{R}{x}\right)} \cdot \dfrac{\pi \cdot \varepsilon_0}{2 \cdot \ln(1.078)}}{\dfrac{\pi \cdot \varepsilon_r}{2 \cdot \ln\left(\frac{R}{x}\right)} + \dfrac{\pi \cdot \varepsilon_o}{2 \cdot \ln(1.078)}} \tag{9}$$

As the mapped area represents a coaxial line the capacitances C_A and C_B are in parallel so they add arithmetically and from equation 4 b and 9,

$$C = \frac{\pi\varepsilon_r}{\ln\left(\frac{R}{x}\right)} + \frac{\dfrac{\pi \cdot \varepsilon_r}{2 \cdot \ln\left(\frac{R}{x}\right)} \cdot \dfrac{\pi \cdot \varepsilon_0}{2 \cdot \ln(1.078)}}{\dfrac{\pi \cdot \varepsilon_r}{2 \cdot \ln\left(\frac{R}{x}\right)} + \dfrac{\pi \cdot \varepsilon_o}{2 \cdot \ln(1.078)}} \tag{10}$$

and because each conductor has been mapped into a coaxial system, then from equations 1 and 4a and disregarding at this stage the effect of the second conductor the self-potential constant r_{11} is given as,

$$r_{11} = \frac{1}{\dfrac{\pi \cdot \varepsilon_r}{\ln\left(\frac{R}{x}\right)} + \dfrac{\dfrac{\pi \cdot \varepsilon_r}{2 \cdot \ln\left(\frac{R}{x}\right)} \cdot \dfrac{\pi \cdot \varepsilon_0}{2 \cdot \ln(1.078)}}{\dfrac{\pi \cdot \varepsilon_r}{2 \cdot \ln\left(\frac{R}{x}\right)} + \dfrac{\pi \cdot \varepsilon_o}{2 \cdot \ln(1.078)}}} \tag{11}$$

The rest of the field, is confined mainly between the interaction of the two parallel conductors so it will be used solely for the determination of constant r_{12}.

In order to find the effect of the second conductor into the first one, the method of images is implemented on the path between the two conductors.

Initially, the potential on a arbitrary point Z, due to a charge and its image, is given by

$$\Phi = \Phi_+ + \Phi_- = -\frac{(+q)\ln\left(\frac{p_+}{p_{o+}}\right)}{2\pi\varepsilon_{eff}} - \frac{(-q)\ln\left(\frac{p_-}{p_{o+}}\right)}{2\pi\varepsilon_{eff}} \quad (12a)$$

where q is the charge on conductor
-q is the charge of the image
p_+ is the distance of the conductor to an arbitrary point Z
p_- is the distance of the image conductor to arbitrary point Z
p_{o+} is the distance of the conductor to the zero potential point
p_{o-} is the distance of the image conductor to the zero potential point
ε_{eff} is the effective permittivity of the system which can be calculated using the conformal mappings technique of section 3 of this paper.

The shield will simplify the calculations using the fact that is grounded and the point in question could be assumed to be on the surface of circle with radius 2R. Furthermore, it is assumed that the charge on the surface of the inner conductors is uniform.

Applying the superposition theorem on the surface of the shield, on the path between the two conductors and the shield regarding initially the *conductors as point charges*, [4], (for simplicity of calculation the outer conductor will initially considered as spherical, as the geometry examined on this paper is mapped similarly in a conductor case)

$$\Phi = \frac{1}{4\pi\varepsilon_{eff}}\left[\frac{q_o}{p_{o-}} + \frac{q}{p_{o+}}\right] = 0 \quad (12b)$$

The charge of the image conductor has been assigned for clarity as q_o in this case.
Applying equation 12b to the examined system of Figure 1 gives,

$$\Phi = \frac{1}{4\pi\varepsilon_{eff}}\left[\frac{q_o}{\left|R^{'} - b_i\right|} + \frac{q}{\left|R^{'} - K\right|}\right] = 0 \quad (13)$$

where b_i is the distance from the centre of the shield to the image of the second conductor and $R^{'}$ the maximum distance from center of shield to its surface.
By solving equation 13 and then dividing by $R^{'}$ the first part of the equation and by K the second one,

$$\frac{\frac{q_o}{R^{'}}}{\left|\hat{r} - \frac{b_i}{R^{'}}\hat{z}\right|} = \frac{-\frac{q}{K}}{\left|\hat{r}\frac{R^{'}}{K} - \hat{z}\right|} \quad (14)$$

where, $\hat{r}$ is a unit vector and refers to radius R of the shield and $\hat{z}$ is a unit vector and refers to the distances of the conductors and their images to the center of the shield

Next step is to equate the numerators and the denominators of equation 14.
One of the solutions of equation 14 (equating the numerators) gives

$$\frac{q_o}{R^{'}} = -\frac{q}{K} \Leftrightarrow q_o = -q\frac{R^{'}}{K} \quad (15)$$

From equation 12b,

$$q_o = -q\frac{p_{o-}}{p_{o+}} \quad (16)$$

Then, combining the results in equations 12a, 15 and 16, the potential of a conductor (surrounded by a cylindrical shield) and its image in an *arbitrary point Z* is given in equation 17.

$$\Phi = \frac{q \cdot \ln\left(\frac{p_-}{p_+}\right)\left(\frac{K}{R^{'}}\right)}{2\pi\varepsilon_{eff}} \quad (17)$$

where p_- is the distance of the image to the arbitrary point Z
p_+ is the distance of the conductor to the arbitrary point Z
q is the charge of the conductor

If the denominator equality of equation 14 is considered then,

$$\hat{r} - \frac{b_i}{R^{'}}\hat{z} = \frac{R^{'}}{K}\hat{r} - \hat{z} \quad (18)$$

Squaring equation 18 gives,

$$1 - 2\frac{b_i}{R^{'}}(\hat{r}.\hat{z}) + \left(\frac{b_i}{R^{'}}\right)^2 = 1 - 2\frac{R^{'}}{K}(\hat{r}.\hat{z}) + \left(\frac{R^{'}}{K}\right)^2 \quad (19)$$

By equating factors of the left hand and right hand sides of equation 19,

$$b_i = \frac{R^{'2}}{k} \quad (20)$$

So equation 17 gives,

$$\Phi_1 = \frac{q_2 \cdot \ln\left(\frac{\overline{A_1B_2}}{\overline{A_1A_2}}\frac{K}{R^{'}}\right)}{2\pi\varepsilon_{eff}} \quad (21)$$

where $\overline{A_1B_2}$ is the distance from the first conductor to the image of the second, $\overline{A_1A_2}$ is the distance between the two conductors.
Using 20 into 21 and substituting the geometrical value of R',

$$\Phi_1 = \frac{q_2 \cdot \ln\left(\frac{b_i + K}{2 \cdot K} \frac{K}{2 \cdot R}\right)}{2\pi\varepsilon_{eff}} \tag{22}$$

and from equation 1,

$$r_{12} = \frac{\Phi_1}{q_2} = \frac{\ln\left(\frac{\left(\left(\frac{2R}{K}\right)^2 + 1\right) \cdot K}{4R}\right)}{2\pi\varepsilon_{eff}} \tag{23a}$$

Because of the symmetry of the structure and the reciprocity being present

$$r_{11} = r_{22}\ ,\ r_{12} = r_{21} \tag{23b}$$

so there is no need for extra calculations for r_{22} and r_{21}.

2.2 Conductor to shield capacitance

However the system of Figure 1 cannot be treated solely with coaxial theory, so the following procedure is required. In order to calculate the capacitance the potential on each conductor have to be related to both individual charges. By inverting equations 1 and 2,

$$q_1 = \beta_{11} \cdot \Phi_1 + \beta_{12} \cdot \Phi_2 \tag{24}$$

and

$$q_2 = \beta_{21} \cdot \Phi_1 + \beta_{22} \cdot \Phi_2 \tag{25}$$

where,

$$\beta_{11} = \frac{r_{22}}{\Delta} \quad \beta_{12} = -\frac{r_{12}}{\Delta} \tag{26a}$$

$$\beta_{21} = -\frac{r_{21}}{\Delta} \quad \beta_{22} = \frac{r_{11}}{\Delta} \tag{26b}$$

and

$$\Delta = r_{11}r_{22} - r_{12}r_{21} \tag{27}$$

But,

$$\Phi_2 = -(\Phi_1 - \Phi_2) + \Phi_1 = -V_{12} + V_{10} \tag{28}$$

Combining equations 24 and 28,

$$q_1 = (\beta_{11} + \beta_{12}) \cdot V_{10} - \beta_{12} \cdot V_{12} \tag{29}$$

But from electrostatic theory [5], for the geometry considered,

$$q_1 = C_{11} \cdot V_{10} + C_{12} \cdot V_{12} \tag{30}$$

Hence,

$$C_{11} = \beta_{11} + \beta_{12} \tag{31}$$

So by solving equation 31 the conductor to shield capacitance can be calculated.

2.3 Mutual capacitance

The next step is to calculate the mutual capacitance between the parallel conductors. In this case conformal mappings will be used as they can map satisfactory any inhomogeneities between the parallel conductors. The conformal mappings used in [6] for the determination of the capacitance of unshielded cables was used.

w to z plane transformation

$$w = x \cdot \sinh x_1 \coth\frac{z}{2} \tag{32}$$

z to w plane transformation

$$z = \ln\frac{w + R\sinh x_1}{w - R\sinh x_1} \tag{33}$$

$$x_1 = \cosh^{-1}\left(\frac{K}{x}\right) \tag{34}$$

The characteristic of this transformation is that the boundaries of the metal conductors in the w plane are transformed in parallel lines in the z plane and the dielectric boundaries into curved contours respectively. The shield is mapped in the area between the parallel lines as in Figure 3b. Conformal mapping on the boundaries is valid because it retains the angle of refraction of lines of force on the boundary [6]. The mutual capacitance is calculated by considering the parallel lines as plates of capacitors and obtaining the capacitance in the area where the shield is not present.

Figures 3a and 3b shows an example of a cable with insulation to inner conductor radius ratio equal to 3. Figure 3a shows a replica of Figure 1 represented on the w-plane for the above ratio. Figure 3b shows the corresponding mapping on the z-plane when equations 33 and 34 are used. As it can be seen the parallel straight lines are mapped at x = ±1.76, as expected from equation 34.

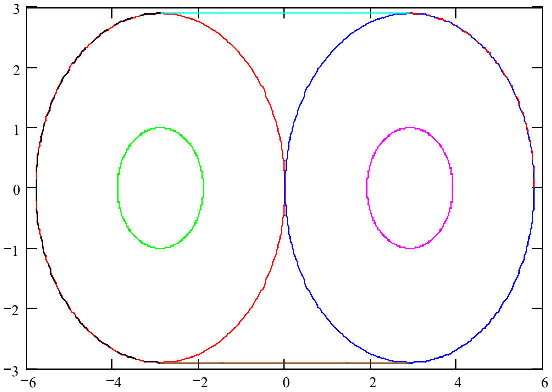

Figure 3a: Cotangent hyperbolic transformation (w-plane)

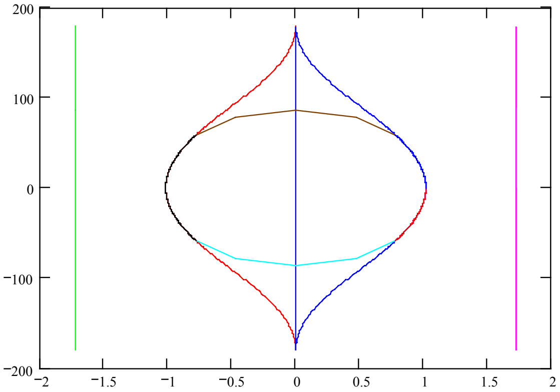

Figure 3b: Cotangent hyperbolic transformation (z-plane)

3. Effective permittivity

This requires calculation of the capacitance (using conformal mappings as in section 2) with and without the dielectric present and then using the ratio of these capacitances to find the effective permittivity. The capacitance required in this step, is calculated by adding arithmetically the conductor to shield and conductor-to-conductor capacitances.

$$\varepsilon_{eff} = \frac{C_{11} + C_{12}}{C_{01} + C_{02}} \qquad (35)$$

where C_{12} is the mutual capacitance calculated in section 2.3, C_{01} is the conductor to shield capacitance for air medium between them, and C_{02} the mutual capacitance between the inner conductors separated by air medium as well.

4. Impedance calculation

Usually the cable manufacturers specify several nominal parameters for the cable. Two of them are the characteristic impedance and the velocity of propagation. Furthermore the dielectric constant of the medium (insulation) ε_r is related to the velocity of propagation, v, by [7],

$$v = \frac{v_o}{\sqrt{\varepsilon_r}} \qquad (36)$$

where v_o is the velocity of light. So when the velocity of propagation is known the value of the medium permittivity can be calculated.

By calculating the effective permittivity in the previous section, then the line can be treated as homogeneous with the permttivity value the effective one.

Then, for a TEM mode of propagation and a homogeneous medium, the following equation states,

$$LC = \mu\varepsilon_{eff} \qquad (37)$$

where L is inductance, C is the capacitance and μ the permeability of free space.

Also,

$$Z = \sqrt{\frac{L}{C}} \qquad (38)$$

Then, the impedance can be given as function of the capacitance C by

$$Z = \frac{\sqrt{\varepsilon_{eff}}}{C \cdot v_o} \qquad (39)$$

The final step is to take into account the effect of the twist on the actual length of the cable and consequently on the capacitance per unit length (which in this case will be 1 meter).

Considering that the voltage and the current propagate along the wires of length L, Pythagoras' theorem gives, the propagation constant β' in the x-axis is given by

$$\beta^{'} = \frac{\beta}{\sin\alpha} = \beta\frac{\sqrt{P^2 + (\pi D)^2}}{P} \tag{40}$$

Where P is the Lay length

D is the diameter of the pair

So by assigning a constant AD as a correction factor ,

$$AD = \frac{\sqrt{P^2 + (\pi D)^2}}{P} \tag{41}$$

Then the total length of the cable Lα, will be given by

$$La = Lt \cdot AD \tag{42}$$

5. Results

A PimF cable has been examined in this paper with the following geometrical characteristics.

Conductor diameter: 0.574mm

Insulation diameter: 1.52mm

Dielectric constant of insulation (foam) = 1.93

The twist lay length for the above calculation was assumed to be 20mm which is a characteristic value for this kind of cables.

Table 1: Comparison between theory and measurements

	theory	Measurements
Capacitance to shield (pF/m)	67	69
Mutual capacitance (pF/m)	45	43
Characteristic Impedance (Ω/m)	102	~100

6. Conclusions

A new method of the calculation of the impedance for cables with non-ideal shields has been developed. The advantage of this method is that it can calculate the capacitance for any shields without employing complex modeling techniques such as the Method of Moments (MOM), Finite Element Analysis Method (FEA) etc. Also the use of conformal mappings involves simple calculations (it maps a complex structure to a much simpler one) and have been integrated in the equations given for the conductor to shield and mutual capacitances. The results presented demonstrate the accuracy of the technique.

7. Acknowledgments

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8. References

[1] ISO/IEC 11801 standard, edition 1.2,January 2000.

[2] M. M. Al-Asadi, A. P. Duffy, K. Hodge, A. J. Willis "*Analysis and modelling of Structured Wiring Communication Channels*", IEE Proc.-Sci. Meas. and Techn., 148(3), pp 129-136(May 2001)

[3] P.A.Laura, L.E.Luisoni, "*Approximate Determination of the Characteristic Impedance of the Coaxial Line System Consisting of a Regular Polygon Concentric with a Circle*", IEEE Trans. on MTT, 160-162.(Fe. 1977)

[4] M.H.Nayfeh, M.K.Brussel, "*Electricity and Magnetism*", John Wiley and Sons, New York, (1985).

[5] S.Zhang " *Calculation of the partial Capacitance in an system of conductors within the Calculable Resistor*" IEEE Trans. on Instrum. and Meas.,43,(6), 929-932.(Dec 1994)

[6] B.N Das, S.B Chakrabarty, and S.R.Rao, "*Capacitance of Transmission Line of Parallel Cylinders in the Presence of Dielectric Coating"*, IEEE Trans. on EMC, 37, (1), 94-96 (Feb1995).

[7] C.R.Paul, "*Introduction to Electromagnetic Compatibility*", J.Wiley and Sons,New York, (1992).

Author biographies

Alexandros Gavrilakis can be found in "Determining Cable Shield Behaviour" in these proceedings.

Muhammed Al-Asadi can be found in "Input Impedance of irregular cascaded systems" in these proceedings

Alistair P Duffy, Kenneth G. Hodge and Arthur J. Willis can be found in "Technology Forecasting Techniques in Communications" in these Proceedings