# Input Impedance of Irregular Cascaded Systems

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## Abstract

In this paper, we report and describe the derivation of an analytical approach for the calculation of the input impedance of a non-identical cascaded network of ‘n’ discrete components based on knowing the propagation constant and the characteristic impedance of each component. These parameters may be obtained analytically or experimentally in a complex form. The developed equation allows the study of the effects of network irregularity, load impedance mismatching and the mismatching of the impedance of cascaded segments of the whole network. It is then applied to calculate the impedance of a multi-segment communication channel at a range of frequencies and validated against a standard equation.

## Keywords

Cascaded Networks; Communication Channels; Communication Cables; Characteristic.

## 1. Introduction

Many constrains may be added to designers and engineers working on high frequency circuits, such as VLSI circuits [1] and communication channels, with the increasing demand for high frequency communication channels to cope with very high data rate transfer, and high frequency circuits. Hence, the effect of impedance mismatches between elements of the cascaded network and the mismatching of the channel termination on the performance of such networks and circuits needs to be investigated.

Communication channels may include very long cables, in which case the impedance of the cable measured at any point along the cable is not constant and depends on local variations of dimensions, such as tolerance in copper diameter and dielectric thickness, and material properties. Figure 1 illustrates the measured characteristic impedance along a cable as a function of the distance between the near end of the cable and the point of measurement. Measurement of the characteristic impedance of three different cables of 10 m length are shown. This longitudinal variation has a great effect on the overall system parameters, including the input impedance and hence on the overall performance of the cascaded system or channel. In addition to that, the use of connectors of different impedance values and different propagation constant produces an unethical performance of the communication system.

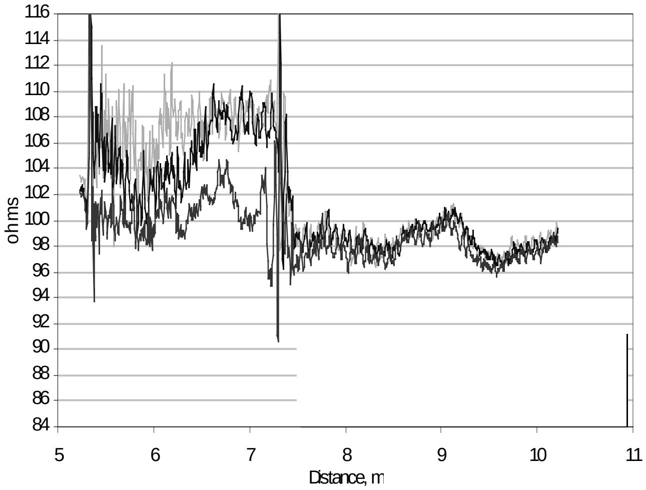

**Figure 1. Measured Characteristic impedance of 3 different cables of 10 m length.**

Previous work [2] presented the calculation of the overall return loss of a communication channel consisting of many cascaded segments and the calculation of the overall attenuation losses of cascaded systems [3]. In this paper, the work has been extended to develop a similar approach for the prediction of the input impedance, $Z_i$, of a network containing any number of elements, $n$. The developed equation is also a function of the impedance of every element, $Z_o$, the physical length of the element, $l$, the propagation constant, $\gamma$ and the frequency of the transmitted signal, $f$.

As the developed equation relies only on the impedance and propagation constant values of the cascaded network elements, it can be used for analysis of channels containing any type of cables such as, STP, UTP and FTP cables. Since the new equation predicts the overall input impedance of any cascaded system, it can also be used for the prediction of the overall return loss and structural return loss of such cascaded system when connected to a source with known source impedance. The developed equation is then used for the prediction of the input impedance and the return loss of different cascaded systems. Results obtained are compared with those obtained using a traditional (but more cumbersome) analytical solution. These comparisons confirm the validity of the derived equations. The next section discusses the traditional approach of calculating the overall input impedance of cascaded systems.

## 2. Traditional Approach

A regular (perfect) cable of impedance, $Z_{o1}$, may be terminated at the far end with a load of impedance, $Z_l$, as shown in Figure 2, where $l_1$ and $\gamma_1$ are the physical length in meter and the propagation constant of the cable respectively.

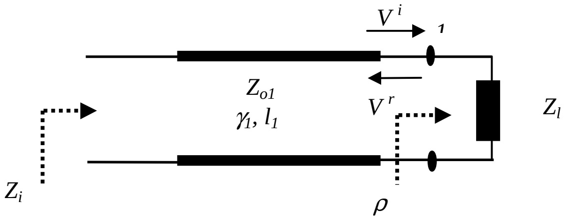

**Figure 2. Schematic diagram of a single cable of impedance $Z_{o1}$ terminated with load $Z_l$**

For such a simple channel the input impedance, $Z_i$, of the channel is given using a traditional approach as described in [4] as:

$$Z_i = Z_{o1}\left[\frac{Z_l\cosh(\gamma_1 l_1) + Z_{o1}\sinh(\gamma_1 l_1)}{Z_o 1\cosh(\gamma_1 l_1) + Z_l\sinh(\gamma_1 l_1)}\right] \quad (1)$$

The input impedance of Figure 1 can also be obtained in term of the reflection coefficient, $\rho = (V^r / V^i)$, at the load end as:

$$Z_i = Z_{o1}\left[\frac{1 + \rho \cdot e^{-2\gamma_1 l_1}}{1 - \rho \cdot e^{-2\gamma_1 l_1}}\right] \quad (2)$$

where $V^r$ *and* $V^i$ are the reflected and the incident voltages at the load end respectively. In this case the reflection coefficient can be obtained as function of both load impedance and cable impedance as:

$$\rho = \frac{Z_l - Z_{o1}}{Z_l + Zo_l} \quad (3)$$

The reflection coefficient given in equation 3 can either be measured or calculated using the complex form of the impedance values. For simple and regular communication channels, this operation is straightforward. The difficulty arises when a number of other elements such as connectors and more than one cable of different characteristic impedance, as illustrated in figure 1, are present in the network. It will also be more difficult if one or more of those connectors and cables is suffering irregularity or deformation as a result of, tolerance in dimensions, production process or bad handling effect [5]. In this case the new developed approach described in this paper is more appropriate for the calculation of the input impedance of such system.

## 3. New Approach Derivation

A general cascaded system containing, *n*, elements can be illustrated using the two port network principles, as shown in Figure 3.

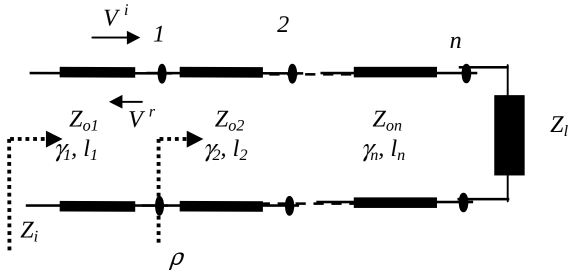

**Figure 3. Schematic diagram of a cascaded network containing *n* segments and terminated with load.**

The input impedance of such circuit can be obtained using equation 1. In this case, looking to the right side of the network input impedance at the connection (*n-1*) between the load and the segment connected to the load can be obtained. This value should be considered as a load connected to the rest of the circuit at connection node (*n-2*). Repeating this process for as many segments as included in the network, the overall input impedance of the cascaded system can be obtained.

Another approach is to measure the reflection coefficient at the connection point (*1*) between the 1st and the 2nd segments of the network. The obtained value can then be used to calculate the input impedance of the network using equation 2. Due to many difficulties that may interfere with the measurement process and the measuring devices, we propose our approach to calculate the reflection coefficient as function of impedance and propagation coefficient values of all cascaded segments of the network.

Following a similar approach described in [2] and [3] and using the relations between incident, transmitted and reflected voltages at the connection between segments along the network, the reflection coefficient at the connection between both 1st and 2nd segments of the network can be obtained as in equation 4.

Substituting equation 4 in equation 2, the overall input impedance of the whole network can then be given as in equation 5.

$$\rho = \frac{Zo_2 - Zo_1}{Zo_2 + Zo_1} + \sum_{k=2}^{n}\left[\left\{\prod_{i=2}^{i=k}\left(\frac{2\cdot Zo_i}{Zo_i + Zo_{i-1}}\cdot\frac{2\cdot Zo_{i-1}}{Zo_i + Zo_{i-1}}\right)\right\}\cdot e^{-2\left\{\sum_{j=2}^{k}\gamma_j l_j\right\}}\cdot\frac{Zo_{k+1} - Zo_k}{Zo_{k+1} + Zo_k}\right] \tag{4}$$

$$Z_i = Zo_1\left[\frac{1+\left\{\frac{Zo_2 - Zo_1}{Zo_2 + Zo_1} + \sum_{k=2}^{n}\left[\left\{\prod_{i=2}^{i=k}\left(\frac{2\cdot Zo_i}{Zo_i + Zo_{i-1}}\cdot\frac{2\cdot Zo_{i-1}}{Zo_i + Zo_{i-1}}\right)\right\}\cdot e^{-2\left\{\sum_{j=2}^{k}\gamma_j l_j\right\}}\cdot\frac{Zo_{k+1} - Zo_k}{Zo_{k+1} + Zo_k}\right]\right\}\cdot e^{-2\gamma_1 l_1}}{1-\left\{\frac{Zo_2 - Zo_1}{Zo_2 + Zo_1} + \sum_{k=2}^{n}\left[\left\{\prod_{i=2}^{i=k}\left(\frac{2\cdot Zo_i}{Zo_i + Zo_{i-1}}\cdot\frac{2\cdot Zo_{i-1}}{Zo_i + Zo_{i-1}}\right)\right\}\cdot e^{-2\left\{\sum_{j=2}^{k}\gamma_j l_j\right\}}\cdot\frac{Zo_{k+1} - Zo_k}{Zo_{k+1} + Zo_k}\right]\right\}\cdot e^{-2\gamma_1 l_1}}\right] \tag{5}$$

where, $Zo_k$ is the impedance of the $k^{th}$ segment and, where $k = n$, $Zo_{k+1}$ is the load impedance, $Z_l$. It should be mentioned here that replacing the load impedance by an open circuit or short circuit, the reflection coefficient of equation 4 can be obtained, and hence the input impedance of the system under such termination conditions can be obtained. Using both calculated values of the input impedance under short and open circuit conditions, the input impedance of the network under a matching load condition can also be obtained using the following equation:

$$Z_i = \sqrt{Z_{ioc}\cdot Z_{isc}} \tag{6}$$

where, *Zioc* and *Zisc* are the input impedance values of the overall network obtained under open circuit and short circuit load termination respectively.

## 4. Equation Validation

To validate the developed equation, the input impedance of seven segments communication channel connected as; {connector 1 (1*cm*), patch cable 1 (1*m*), connector 2 (1 *cm*), horizontal cable (10*m*), connector 3 (1*cm*), patch cable 2 (1*m*), and connector 4 (1*cm*)} is calculated. All the segments have the same dimensions and hence resulting in similar characteristic impedance. The network is connected to matching source impedance and terminated with matching load. The dimensions of the cascaded cables are:

$D$ = 0.94 *mm* is the distance between the centres of the cable conductors.
$r$ = 0.53 *mm* is the radius of each cable conductor.

These cable dimensions resulted in an impedance value of 96.7 $\Omega$. The input impedance of this network was also calculated using the traditional approach of equation 1. Both amplitude and phase of the impedance values are plotted in figures 4 and 5 respectively as function of the frequency of operation $f$. Both figures illustrate an excellent agreement between both sets of results obtained using the traditional approach and the new developed approach.

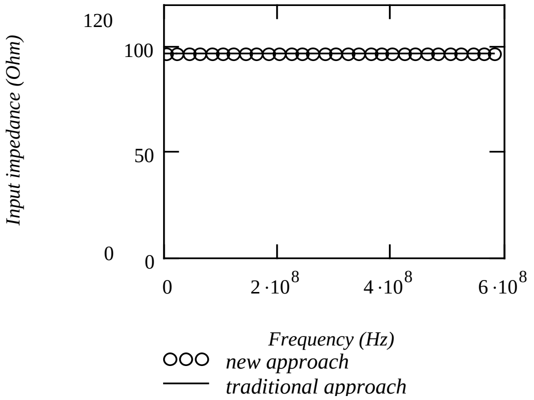

**Figure 4. Input impedance of 7 segments network validated against the traditional approach of equation 1.**

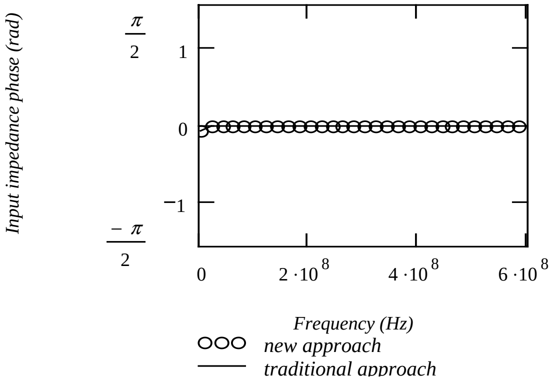

**Figure 5. Phase of the impedance illustrated in figure 4**

## 5. Application and Results

To illustrate the performance of the new equation, it is used for the calculation of the input impedance of irregular 7-segment system discussed earlier. To simplify the calculations, circular cross section connectors are used. Dimensions of each connector are; $D$ = 0.85 *mm* and $r$ = 0.46 *mm*, while dimensions of each patch and horizontal cable are; $D$ = 0.87 *mm* and $r$ = 0.55 *mm* is the radius of each cable conductor. The length of each conductor is 1 *cm*, the length of each patch cable is 1 *m* and the length of the horizontal cable is 10 *m*.

The lay length of each cable is 20 *mm* and is taken into account for the calculation of the input impedance. Those dimensions resulted in impedance of 100.7 $\Omega$ of each connector and 84.93 $\Omega$ of each cable. The system is connected to a source of 100 $\Omega$ impedance and terminated with similar load impedance. Both amplitude and phase of the input impedance of overall system is calculated and plotted as function of the frequency of operation as illustrated in figure 6 and 7 respectively.

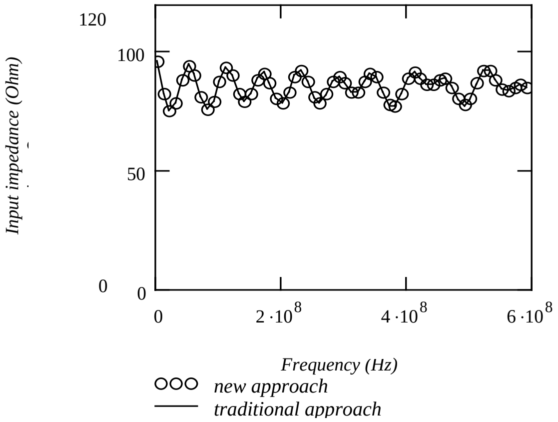

**Figure 6. Impedance amplitude of a 7 segment irregular system calculated using both traditional and new approach.**

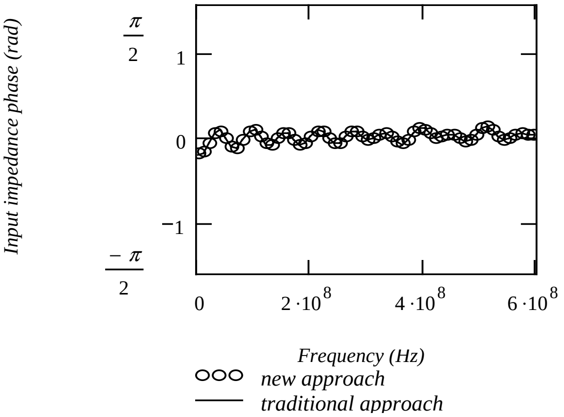

**Figure 7. Phase of the impedance illustrated in figure 6.**

Further validation can be achieved by using the new equation for the calculation of the input impedance using open circuit terminations. Both traditional and new approaches are used to validate the developed equation. Again amplitude and phase of the input impedance are calculated and plotted as function of the frequency of operation as illustrated in figures 8 and 9 respectively.

Furthermore, the new equation is used for the calculation of the input impedance of irregular system of a 100 m horizontal cable, using both equation 6 where short and open circuit terminations are used and using a matched load termination. Results are obtained for the same dimensions described earlier. Amplitude and phase are both calculated and plotted as illustrated in figures 10 and 11 respectively.

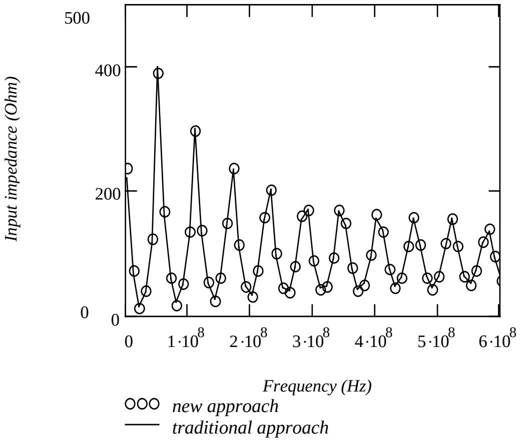

**Figure 8. Reproduction of figure 6 for an open circuit termination.**

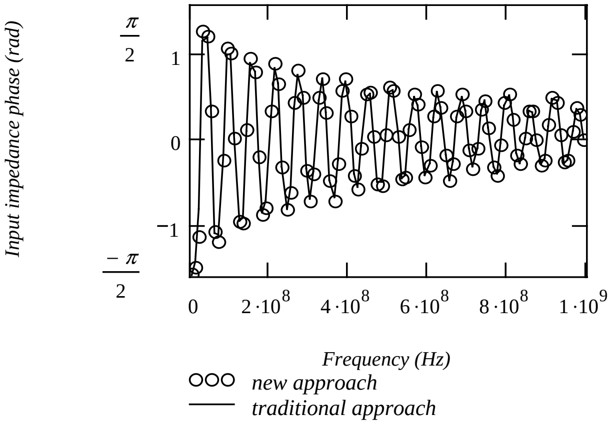

**Figure 9. Phase of the impedance illustrated in figure 8.**

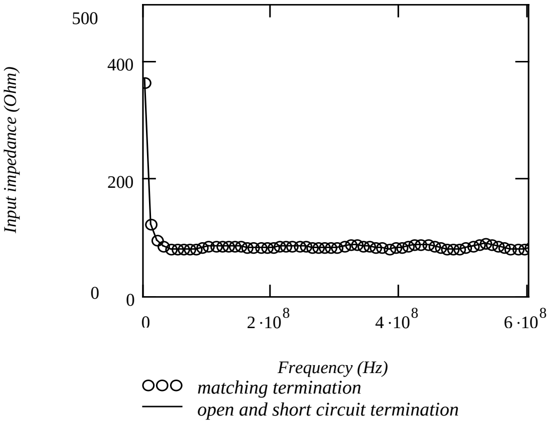

**Figure 10. Amplitude of the input impedance where a 100-m cable is used obtained using the new equation and using matching and open and short circuit terminations.**

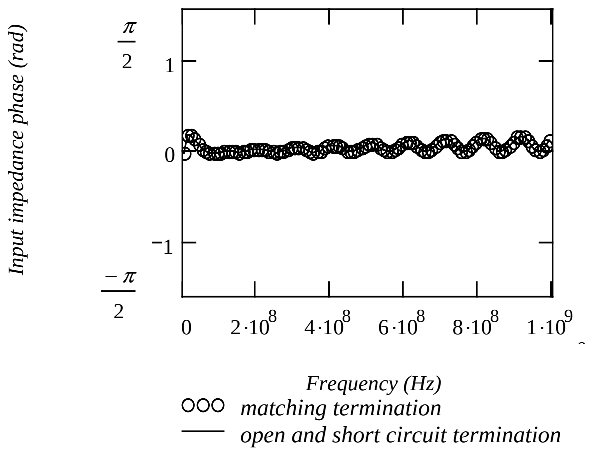

**Figure 11. Phase of the impedance illustrated in figure 10.**

Results obtained here illustrate the need to the development of such effective approach rather than using an iterative approach that can be computationally costly as the number of segments in the cascaded system is getting bigger. It should also be mentioned here that the input impedance obtained here can be used for the calculation of the return loss of the system using equation 28 of reference [2].

## 6. Conclusions

The calculation of the input impedance of any cascaded network, such as communication channel, can be achieved using an efficient and very simple equation presented here. The developed equation allows the direct calculation of input impedance without requiring the iterative calculations of current traditional equation (1). The new equation (4) can readily provide the reflection at any interface along the cascaded network. The performance of the new equation is illustrated by applying it to different illustrative channels with different terminations, different dimensions and different length of the horizontal cable. Excellent agreement was achieved between results obtained using the traditionally used approach and the new developed equation. The new equation performs equally well with any number of segments and at any frequency of operation. It also performs well for any load termination and can be applied to any structured cabling installation containg UTP, FTP and STP cables.

## 7. References

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## Authors Biographies

**Dr. Mohammed Al-Asadi** received his BSc. degree from the University of Basra, Iraq and an MPhil degree from the University of Nottingham, UK, both in Electronic and Electrical Engineering in 1988 and 1992 respectively. Between 1993 and 1996 he was working on a research project at the University of Nottingham, developing a program to incorporate charge particle motion in electromagnetic fields with transmission line modelling method.

In 1996 he joined De Montfort University in Leicester, UK, where he is working as a Research Fellow and where he obtained his PhD degree. His research involves the investigation of high frequency performance of structured wire cabling in communication systems. He is the author and co-author of many journal and conference papers. His research interests include EMC, communication cables, electron beam, field-particle interaction devices, numerical methods and simulation of electromagnetic fields. He is currently working on the Bit Rate Error Correction in wireless mobile communication.

**David A. Jackson** holds a BSc(Hons) in Electrical and Electronic Engineering from Abertay University, Dundee, Scotland. His career has been dominated by the design of test systems, initially as a Research and Development engineer working on test facilities for microprocessor systems. He has spent over 16 years in the cable industry designing and implementing radio frequency test methods.

For biographies of **A. P. Duffy, K. G. Hodge and A. J. Willis,** please see paper "Technology Forecasting Techniques in Communications" presented in this symposium.